Simulation of tidal effects on groundwater flow and salt transport in a coastal aquifer with artificial drains (Pressure Equilibrium Modules)

Peter Engesgaard
Department of Geography and Geology
University of Copenhagen

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1. Introduction and objectives

The results of a numerical study of groundwater flow and the movement of salt in a coastal aquifer are reported. The research objective has been to numerically investigate the effects of Pressure Equilibrium Modules (PEMs) on groundwater flow in a coastal aquifer, especially their draining effect. A number of simulation scenarios have been investigated representing different flow systems. Furthermore, a small sensitivity study was carried out looking into the effects of the permeability contrast between the beach and a higher- or lower permeable layer in the beach and how the connectivity of this layer to the sea on how PEMs affect draining.

2. Model setup

Different flow systems have been investigated where freshwater inflow to the beach is varied from almost no inflow to a very high inflow. The base case is developed to approximate the conditions at Holmsland at the West coast of Denmark. For all systems the effects of having a gravel layer present were simulated with and without PEMs. Additional simulations studies were carried out having one or two layers of contrasting permeability (gravel to clay) present or not. The effect of disconnecting the gravel layer with the sea was also investigated.

The conceptual simulation model is a two-dimensional cross-section of the beach system with or without PEMs. The width of the cross-sections is assumed to be 1 m.

2.1 2D Flow and mass transport models

The flow and salt transport models assume the following:

- 2D variably-saturated groundwater flow and density-dependent transport of salt
- The hydraulic properties of the unsaturated zone are described by the van Genuchten equations relating soil moisture with hydraulic tension and conductivity
- The flow equations are formulated in terms of fresh water head
- The PEMs are simulated as flow pipes (Hagen-Poiseulle flow).

The set of equations for 2D variably-saturated flow and density-dependent transport of salt are well-known and will not be given here. A description of how to incorporate the pressure equilibrium modules are given below. The simulation code FeFlow was used (Diersch, 2006).

2.2 Including Pressure Equilibrium Modules as drains

The pressure equilibrium modules are inserted as so-called discrete feature elements in the model. Flow in the PEMs is assumed to follow axi-symmetric Hagen-Poiseulle flow in a pipe of radius R (pure translation of flow and no inertial effects). This means that flow can be described as an equivalent to Darcy flow according to

\[
q = K_f \left( \frac{\partial h}{\partial z} + \frac{\rho - \rho_o}{\rho_o} \right)
\]

(1)
where \( f_\mu \) is the ratio of the viscosity of freshwater (\( \mu_o \)) and saltwater (\( \mu \)). The density of freshwater and saltwater is given as \( \rho_o \) and \( \rho \), respectively. The ratio \( (\rho - \rho_o) / \rho_o \) is called the density ratio. The equivalent hydraulic conductivity \( K \) for a pipe is:

\[
K = \frac{r_{\text{hydr}}^2 \rho_o g}{2\mu_o} = \frac{R^2 \rho_o g}{8\mu_o}
\]  

(2)

where \( r_{\text{hydr}} \) is the hydraulic radius of a pipe (\( R/2 \)). Thus, by specifying the radius it is possible to simulate flow in discrete elements (pipes) embedded in the porous matrix elements. For example, with \( \rho_o=1000 \text{ kg/m}^3 \), \( \mu_o=1.3\times10^{-3} \text{ Pas} \), and \( g=9.81 \text{ m/s}^2 \) one can calculate an equivalent hydraulic conductivity of about 1500 m/s (1.3*10^8 m/day) for a pipe with radius of 0.04 m. This is a factor of 5 million higher than the hydraulic conductivity of a sandy porous medium (like what is used for a beach in the current study). Thus, the pipes are very conductive to flow.

Since the model is two-dimensional this also means that the width of the model is 1 m. It is implicitly assumed that the pipes fill out all of the width, which is of course not true. In fact, for the current system a pipe only occupies 0.08 m per 1 m width.

2.3 Beach geometry, boundary conditions and choice of parameters

Figure 1 shows the idealized model setup of a beach system. The section is 100 m long with a sloping beach face from -2 m to +2 m. The amplitude of the tide is assumed to be \( A=0.5 \text{ m} \), Figure 2. The high and low water mark (HWM and LWM) are indicated on Figure 1. Mean sea level (MSL) is 0 m and cuts the beach at the location \( x=20 \text{ m} \).
The boundary conditions are as follows:

- The red line indicates the section where a tidal boundary condition (Figure 1) is applied. However, for each nodal point the tide is adjusted to provide the equivalent freshwater head according to elevation. The details of this will not be given here.
- A seepage face is also specified between LWM and HWM, which means that it is assumed that the water table follows the beach for example when going from high to low tide. When the seepage face is active, only outflow is allowed. This is incorporated in the model using so-called boundary constraints. It means that the water table can detach from the beach when going from high to low tide.
- The rest of the upper boundary is assumed to be a water table with zero influx.
• The bottom is assumed impermeable giving a variable thickness of the coastal aquifer (8-12 m).
• The right boundary (blue line) is a fixed head boundary condition (head of 0.3 m). The choice of a head of 0.3 m is based on a calculation of how much water should discharge to the coast from the strip of land separating the ocean and Ringkøbing Fjord. With a precipitation of around 800-1200 mm/year, assuming that 50% infiltrates in the sand dunes, and taken the half-width of the length of the strip of sand (650 m) one can calculate an approximate discharge of 0.9 m$^2$/day. When the tide is at MSL there is a head gradient of 0.3-0.0 m from the freshwater boundary to the point where the MSL cuts the beach. From Darcys law one can compute a freshwater inflow of about 1.7 m$^2$/day. However, as Nielsen (1990) pointed out there will be a water table overheight. Without showing the details here (see Engesgaard, 2006) the overheight amounts to about 0.25 m, although Nielsens analytical solution is not strictly valid for the selected system. Thus, the inflow will be reduced. The mean inflow over a tidal cycle for a homogeneous beach has later been simulated to be approximately 0.65 m$^2$/day. Including a gravel layer and PEMs will increase this average inflow because of a higher permeability of the beach.

An approximate 0.5 m thick gravel or clay layer can be present in the beach. This layer may connect to the sea or not. The PEMs are modelled as pipes and their positions are also indicated in the figure. It is only the 1 m slotted screen of the PEMs that have been included.

The model parameters are given in Table 1. It is assumed that the sand in the beach has a hydraulic conductivity of 25 m/day and that the gravel layer has a hydraulic conductivity that is a factor of 10 higher. This layer will be also be simulated as a silt/clay layer with a hydraulic conductivity a factor of 10-100 less than that of the sand. The discretization is 1 m in the horizontal direction (100 elements). In the vertical direction 25 elements have been used given a variable mesh length.

The PEMs were located at $x=9$ m, 19 m, 29 m, and 39 m and 0.5 m below the beach face. In the experiments reported by Engesgaard (2006) the screens of the PEMs were located approximately 0.8 m below the beach face, however, due to the choice of discretization in the vertical direction.
(0.5 m) it was not possible to exactly match this position. The effects of locating the PEMs one meter deeper is investigated.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
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<tbody>
<tr>
<td>Saturated hydraulic conductivity for sand, (K_s=K_z)</td>
<td>25)</td>
<td>m/day</td>
</tr>
<tr>
<td>Saturated hydraulic conductivity for gravel, (K_c=K_z)</td>
<td>250 (0.25, 0.025)</td>
<td>m/day</td>
</tr>
<tr>
<td>Porosity, (n)</td>
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<td>-</td>
</tr>
<tr>
<td>van Genuchten max. saturation</td>
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<td>-</td>
</tr>
<tr>
<td>van Genuchten min. saturation</td>
<td>0.0025</td>
<td>-</td>
</tr>
<tr>
<td>van Genuchten fitting coefficient (A)</td>
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<td>1/m</td>
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<tr>
<td>van Genuchten fitting exponent (n)</td>
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<td>-</td>
</tr>
<tr>
<td>Longitudinal dispersivity</td>
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<td>m</td>
</tr>
<tr>
<td>Transverse dispersivity</td>
<td>0.02</td>
<td>m</td>
</tr>
<tr>
<td>Molecular diffusion coefficient</td>
<td>(8.64 \times 10^{-5})</td>
<td>m²/day</td>
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<tr>
<td>Freshwater and saltwater density</td>
<td>1000 and 1029</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Density ratio</td>
<td>0.029</td>
<td>-</td>
</tr>
<tr>
<td>Viscosity</td>
<td>(1.3 \times 10^{-3})</td>
<td>Pas</td>
</tr>
</tbody>
</table>

Table 1: List of parameter values used in model scenarios. The values in parenthesis are values used in different sensitivity studies.

3. Results

All simulations were run to quasi-steady state by repeating the tidal cycle for about 50 days. The results in this section are from one tidal cycle after that.

3.1 General simulation results on flow and salt distributions

Figure 3 shows a close-up view of the simulated water table at low tide (LWM), mean tide (MSL), and high tide (HWM). Recall that the fixed head is 0.3 m at the freshwater boundary. At low tide there is a significant head gradient towards the sea, while at high tide the gradient is opposite, with flow from the sea into the fresher part of the aquifer. The simulated water content above the water table at low tide ranges from 100% saturation at the LWM to about 50% at the HWM.

Figure 4 shows the simulated distribution of salt water at low and high tide for a homogeneous beach, i.e., no gravel/clay layer or PEMs. It is the relative mass fraction that is displayed, where a mass fraction of 1 is salt water and a mass fraction of 0 is fresh water. A secondary plume of saltwater develops at the high water mark. This is in agreement with other findings (e.g. Robinson et al., 2007ab). If the back-ground freshwater flow was higher a freshwater outflow tube could develop between this secondary plume and the saltwater wedge.
Figure 3 Simulated water table at low, mean, and high tide for the homogeneous case.
Figure 4 Simulated mass fraction distribution for homogeneous beach (without gravel/clay layer and PEMs) at low tide (upper) and high tide (lower). A mass fraction of 1 is salt water, a mass fraction of 0 is fresh water.

Figures 5 and 6 show the same results but now plotted with velocities also. At low tide the zone of discharge is close to the LWM with maximum velocities of 1.0 m/d. At high tide there is a rather complicated flow field with both inflow and outflow. Inflow takes place from the LWM and
seaward and around the HWM. In-between there is outflow. Maximum velocities are 0.3 m/d near the HWM.

Figure 5 Close up view of the simulated distribution of mass fraction and velocities at low tide in an open system with no gravel layer or PEMs. The three markers show LWM, MSL, and HWM. The arrows indicate direction and strength.

Figure 6 Close up view of the simulated distribution of mass fraction and velocities at high tide in an open system with no gravel layer or PEMs. The three markers show LWM, MSL, and HWM. The arrows indicate direction and strength.

Figures 7 and 8 show the results with a gravel layer. At low tide, the highest velocities are now found in this layer, but the gravel layer has the effect of providing a slightly more diffuse outflow with maximum velocities now only up to 0.7 m/day. Notice that at high tide the flow field is quite different in that now only inflow occurs primarily through the gravel layer.
Figure 7 Close up view of the simulated distribution of mass fraction and velocities at low tide in an open system with a gravel layer but no PEMs. The three markers show LWM, MSL, and HWM. The arrows indicate direction and strength.

Figure 8 Close up view of the simulated distribution of mass fraction and velocities at high tide in an open system with a gravel layer but no PEMs. The three markers show LWM, MSL, and HWM. The arrows indicate direction and strength.

Figures 9-10 show similar results but now for a system with no gravel layer but 4 PEMs. The location of the PEMs can be seen in Figure 1. Two sets of figures are shown in each figure; top figure shows the saltwater distribution together with two types of velocities; bullets and arrows. The bullets do not show the magnitude of the velocities, only direction. The arrows show magnitude and direction. The bullets are included to show the general flow system, otherwise, because of the high velocities in the PEMs, only these would be visible. The bottom figure shows the freshwater head distribution together with the logarithm to the velocities, i.e., \( \log (\text{velo}) \), plotted then only as a line with an arrow.
At low tide the PEM near the low water mark is most active with very high velocities 180 m/d or approximately 0.2 cm/s. In the bottom figure it looks like the almost horizontal flow velocities are deflected towards the bottom of the PEMs and out of the PEMs near the top. All PEMs show vertical velocities, which seems intuitively correct.

![Close up view of the simulated distribution of mass fraction and velocities (top) and freshwater head and log (velocities) (bottom) at low tide in an open system with no gravel layer but 4 PEMs. The three markers show LWM, MSL, and HWM.](image)

At high tide, Figure 10, it is the PEM near the high water mark that is most active (top figure). It is clear that the PEMs affect the saltwater distribution, see also later. The effect of the PEMs on the velocity distribution is much more apparent at high tide (bottom figure). Again all PEMs show upward flow, despite that flow just outside the PEMs can be downward. The left-most PEM shows a circulation, where flow moves down and into the PEM and then back up through the PEMs. The head distribution around the bottom and top of the PEMs confirm that flow is into the PEM at the bottom and out through the PEM at the top. It is not clear why this circulation comes about. Perhaps a combination of buoyancy effects and flow driven by forced convection. At the middle PEM the
Flow distribution is clearer. Flow diverges upward, enters the bottom of the PEM and exits at the top. At the right most PEM there is again flow down along the PEM, but now part is (apparently) diverted up through the PEM again and parts is flowing towards the landside. Still we see the characteristic head distribution indicating flow into the PEM and out of the PEM. At the beach face flow is outward right at the three PEMs in the tidal zone.

Later it is discussed that numerical oscillations is believed to cause inaccuracies when the tide rolls past a PEM. Thus, this may also have affected the simulated flow fields in the figures above.

Figure 10 Close up view of the simulated distribution of mass fraction and velocities (top) and freshwater head and log (velocities) (bottom) at high tide in an open system with no gravel layer but 4 PEMs. The three markers show LWM, MSL, and HWM.

Figure 11 shows the damping of the tidal signal as a function of distance from the sea. The location of the observation points are shown in Figure 1. The red line is from the point at the low water mark and is identical to the tidal signal (corrected for freshwater head). The dark line is at the mean sea water level and shows the development of the seepage face, where the water table detaches from the
beach. This takes place when the tide reaches the beach level \( h = z = 0 \). Because of the high hydraulic conductivity and the freshwater inflow the tidal signal is quickly dampened and attenuated (peak occurs later more inland).

![Damping of tidal signal](image)

Figure 11 Simulated freshwater head in 6 observations points (see Figure 1). The red and black lines are identical to the applied tidal signal at the LWM and MSL, respectively.

Figures 12 and 13 summarize the simulation results for the salt distribution at low and high tides. In all cases a secondary salt plume may be found at the high water mark. The presence of the gravel layer results in a slightly smaller plume due to the preferential channelling of freshwater through this layer. The presence of PEMs also has an effect on the saltwater distribution. At the bottom of the PEMs the concentration is slightly higher than outside at high tide, while at the top of the PEMs the concentration is slightly lower. The latter is likely due to the preferential upward discharge of water seen in Figure 10.
Figure 12 Simulated salt distribution at low tide for four different cases. The locations of the PEMs are shown with solid dark lines.
Figure 13 Simulated salt distribution at high tide for four different cases. The locations of the PEMs are shown with solid dark lines.

The simulated spatial distribution of the absolute magnitude of the velocity is shown in Figure 14 for the case of no gravel layer with PEMs. The four PEMs are not active in a uniform way. At low tide, it is the PEM near the LWM that is active with upward directed velocities near 200 m/day (0.2 cm/s), while at high tide, it is the corresponding PEM near the HWM that is most active. Between the most active PEMs, the velocities are in the range 0.2 m/day (high tide) to 0.5 m/day (low tide).
Figure 15 shows a close-up view of the velocity distribution at low tide with and without PEMs. Without PEMs the highest flow rates (0.8 m/day), is found near the LWM. With PEMs it is the PEM near the LWM that is most active channelling flow upwards, but notice that flow exits the PEM and is reduced to the same order of magnitude in velocity as without the PEMs.

Figure 14 Simulated magnitude of velocities at low tide (upper) and high tide (lower). Notice, it is the absolute value of the velocity that is plotted, i.e, it does not show direction. Units are in m/day.
Figure 15 Close-up view of velocity distribution at low tide with and without PEMs. Units are in m/day.

Figure 16 shows the tidal changes in the velocities at the mid-point of the four PEMs for the case without a gravel layer. Recall that the PEMs are located near the LWM, MSL, and HWM (one just below and one just above). There are peak flows of up to 0.2-0.3 cm/s in the two PEMs closest to
the sea, whereas the two other PEMs located above the MSL is much more inactive likely because the water table during low tides drops below the PEMs. It is also apparent that the two most active PEMs have peak flows at different times during the tidal cycle. At low tide it is the PEM near the LWM that is most active, and, vice versa, at high tide it is the PEM near the HWM (but below) that is most active.

The simulated velocities at the mid-point of the four PEMs over a tidal cycle are shown in Figure 16. Black line (x=9 m, LWM), red line (x=19 m, MSL), blue line (x=29 m, <HWM), and green line (x=39 m, >HWM). High and low tides are indicated with vertical solid and dashed lines, respectively.

### 3.2 Flow balances

Figure 17 and 18 show the simulated results for an open system without and with PEMs. The freshwater flux is on the order of 2.5 m$^2$/day entering the beach at low tide and -1.0 m$^2$/day leaving the beach at high tide for the homogeneous case without PEMs. These fluxes increase slightly when including a gravel layer and PEMs because of the increased bulk permeability of the aquifer.
Figure 17 Simulated inflow and outflow in beach section (see Figure 1) for an open system with no gravel layer. Top figure, simulations with PEMs are shown with solid lines, simulations without PEMs with dashed lines. Bottom figure, shows differences in inflow and outflow with and without PEMS. High and low tides are indicated with vertical solid and dashed lines, respectively.

The smallest (least negative) outflow occurs during high tide and the highest during low tide. The bottom figure in Figures 17 and 18 show the differences between simulation without and with PEMs for inflow as well as for outflow. In the case of a homogeneous beach (Figure 17) one can note that the in- and outflows are not identical over a tidal cycle, e.g. outflow is clearly different at the two times in between low and high tide. The reason for this is not known precisely, but can be related to numerical difficulties because the boundary condition in the tidal zone is continuously changing from a fixed head boundary to that of an only outflow seepage boundary. This can be the cause of the small (non-symmetric) bumps in the curves for the simulation without PEMS. The simulations with PEMS show greater fluctuations in the in- and outflows. As shown in Figure 3 the water table moves up and down along the PEMS during a tidal cycle. This causes a very complicated flow field around the PEMS (see also Figures 9-10 and 14), which leads to greater oscillations.

Nevertheless, Table 2 shows the integrated in- and outflow for homogeneous case. The in- and outflows represent the area under each curve in Figure 17 (top figure). The differences in flows have been calculated, where a positive difference means that flow is greater with PEMS. A negative flow means less flow with PEMS. For the homogeneous beach the PEMS cause bother greater inflow and outflow. The inflow increases from 1.16 m²/day to 1.25 m²/day, ie. 0.09 m²/day, or about 7%. Outflow increases by 0.21 m²/day or about 10%. So more water flows out than flows in. The difference (0.21-0.09=0.12 m²/day) is approximately 5-10% of inflow/outflow.
<table>
<thead>
<tr>
<th>Description</th>
<th>Inflow (m$^3$/day) – difference in %</th>
<th>Outflow (m$^3$/day) – difference in %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No PEM</td>
<td>With PEM</td>
</tr>
<tr>
<td>Homogeneous (Figure 17)</td>
<td>1.16</td>
<td>1.25</td>
</tr>
<tr>
<td>Gravel Layer (Figure 18)</td>
<td>1.64</td>
<td>1.74</td>
</tr>
<tr>
<td>Gravel Layer w. no connection (Figure 20)</td>
<td>1.37</td>
<td>1.49</td>
</tr>
<tr>
<td>“Silt” Layer 0.1*K (Figure 21)</td>
<td>1.17</td>
<td>1.27</td>
</tr>
<tr>
<td>“Clay” Layer 0.01*K (Figure 22)</td>
<td>0.97</td>
<td>1.34</td>
</tr>
<tr>
<td>2 Gravel layers (Figure 24)</td>
<td>1.95</td>
<td>1.89</td>
</tr>
</tbody>
</table>

Table 2 Summary of simulations showing inflow and outflow.

In the case with a gravel layer the increase in inflow is about the same, however the outflow is now smaller with PEMs than without. Thus, there is a negative influence of the PEMs. These differences are small when compared to the total in- or outflows.
Figure 18 Simulated inflow and outflow in beach section (see Figure 1) for an open system with a gravel layer. Top figure, simulations with PEMs are shown with solid lines, simulations without PEMs with dashed lines. Bottom figure, shows differences in inflow and outflow with and without PEMS. High and low tides are indicated with vertical solid and dashed lines, respectively.

Figure 19 shows the ratio of the sum of the total flows through the 4 PEMs vs. the absolute value of the net inflow and outflow to the sea (magnitude of inflow minus outflow). This is for the case without a gravel layer. The flow through one PEM is simply calculated as $Q=VA$, where $V$ is the mean velocity of the 3 nodes representing a PEM and $A$ is the cross-sectional area of the pipe. The PEMs transport a lot of water, approximately 50% of the net inflow/outflow. Inflow is very low at low tide so here on can say that the PEMs almost transport all of the water eventually discharging to the sea. However, it is important to remember that it is not the same as saying that the PEMs discharge directly to the sea. Instead one can say that water is quickly routed vertically one meter before again being transported much more slowly through the porous media. For example, Figure 15 shows very high absolute velocities in the PEM near the low tide but also increased velocities in the porous medium right above the PEM. At high tide it is much more difficult to interpret the results since, as Figure 10 shows, there is both inflow and outflow.
3.3 Effect of disconnecting the gravel layer from the sea

In this set of simulations the gravel layer is disconnected from the sea by removing the three gravel cells in the top row (see Figure 1), thus, these three cells now consist of sand. Figure 20 shows that this has a negative effect on the draining (outflow) capacity of the PEMs. At low tide the simulations with PEMs discharge less when compared to the case without PEMs. It is not entirely clear why this happens. One explanation can be that at low tide only two PEMs are active, the two most upstream PEMs are now located in the unsaturated zone. The majority of the outflow is captured by the two active PEMs, so instead of discharging directly to the sea over a short distance groundwater flows up in one PEM and then transported laterally in the gravel layer for 5-10 m before again being released to the sand and finally to exit near where the low tide meets the beach.

Table 2 shows that the in- and outflows are reduced, but also that the existence of PEMs enhances inflow but restricts outflow.
3.4 Low-permeable layers

Two sets of simulations were conducted, where the hydraulic conductivity of the layer was decreased to 2.5 m/day and 0.25 m/day, which is a factor of 10 and 100 less than the hydraulic conductivity of the sand.

Figures 21 and 22 show the results of these simulations. The gravel layer is now no longer a “gravel” layer but rather a silt/clay layer that lies as a cap beneath the beach in the tidal zone, where the discharge to the sea mainly occurs. The PEMs therefore will penetrate this capping layer allowing water to access the sea more easily. The figures show that this will both lead to increased inflow and outflow. With a “silt” layer approximately just as much water flows in as flows out, Table 2. With a “clay” layer up to more than 37% now flows in with PEMs, while the increased outflow is much less. The PEMs therefore have a negative effect. One explanation can be the density effect where the pipes are more active.
Figure 21 Simulated inflow and outflow in beach section (see Figure 1) for an open system with a “silt” layer with a $K=2.5$ m/day (factor of 10 less than the $K$ in the sand). Top figure, simulations with PEMs are shown with solid lines, simulations without PEMs with dashed lines. Bottom figure, shows differences in inflow and outflow with and without PEMs. High and low tides are indicated with vertical solid and dashed lines, respectively.

Figure 22 Simulated inflow and outflow in beach section (see Figure 1) for an open system with a “clay” layer with a $K=0.25$ m/day (factor of 100 less than the $K$ in the sand). Top figure, simulations with PEMs are shown with solid lines, simulations without PEMs with dashed lines. Bottom figure, shows differences in inflow and outflow with and without PEMs. High and low tides are indicated with vertical solid and dashed lines, respectively.
inflow and outflow with and without PEMS. High and low tides are indicated with vertical solid and dashed lines, respectively.

3.5 Effect of having several gravel layers connected by PEMS

Figure 23 shows the case with two gravel layers and extended PEMS. These are now 2 meters long. The upper layer is the same as before and connects to the sea. The PEMS connect the two gravel layers.

![Simulated saltwater distribution and flow (high tide) in the case with two gravel layers and PEMS.](image)

Figure 23 shows that both layers are active in transporting water at high tide. It is also clear that the saltwater distribution is affected.

Figure 24 shows the simulated in- and outflow. As before the PEMS both cause extra inflow and outflow. However, this time the net inflow is less with PEMS, while net outflow is increased. The PEMS therefore have a positive effect, albeit very small when compared with the absolute magnitude of in- and outflow.
3.6 Effect of tides

The effects of the tides have been investigated by running the model to steady-state using the open system and a sea level of 0 m corresponding to MSL (however, corrected for freshwater head).

<table>
<thead>
<tr>
<th></th>
<th>Gravel</th>
<th>No gravel</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>No PEMs</td>
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<tr>
<td>Inflow</td>
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</tr>
<tr>
<td>Outflow</td>
<td>-2.01</td>
<td>-2.32</td>
</tr>
</tbody>
</table>

Table 3 Steady inflow and outflow

The presence of a gravel layer increases both inflow and outflow. The PEMs cause a higher inflow and outflow with approximately the same amount. The changes caused by the PEMs are generally high 13-53% but the net result is about zero change in the two systems (i.e., just as much higher inflow as outflow).
3.7 Comparing different scenarios

Figure 25 shows the simulated saltwater fluxes (inflow and outflow, g/day) through the beach. It was not possible to compute water fluxes in this way; however the absolute value of the Darcy fluxes across the beach face is discussed in Section 3.8. The balance only accounts for flow through the nodes which connects to the sea, i.e., from the HWM and seawards. The figure shows three systems; homogeneous beach, a beach with a gravel layer, and a beach with a clay layer.

Figure 25 Simulated inflows (positive) and outflow (negative) through the beach at nodes that connect with the sea. The solid and dashed lines represent simulations with and without PEMS, respectively.
Three things may be noted; (1) at low tide the gravel layer actually results in slightly higher outflow, (2) at the mean tide the PEMs generally results in a slight increase in outflow, and (3) at high tide the PEMs significantly increases inflow. However, the differences between the simulations with and without PEMs are small.

Figure 26 shows the same results but now for the case of a steady system, i.e., no tides. Only the cases with a homogeneous beach and a beach with a gravel layer were simulated.

![Figure 26](image)

Figure 26 Simulated steady inflows (positive) and outflow (negative) through the beach at nodes that connect with the sea. The black and blue lines represent simulations without and with PEMs, respectively.

Again, inflow is positive, and outflow is negative. Remember that MSL cuts the beach at x=20 m. From the case with no PEMs it can be seen that outflow takes place near MSL and is constrained to a zone with a width of about 1-2 m. The effects of the PEMs at x=9 and 19 m are clearly seen resulting in an extra outflow, but also a wider discharge zone, now 2-3 m. However, the PEMs also induce extra inflow. The gravel layer also has an effect mostly by causing an extra inflow where the layer cuts the beach (x=13-16 m). The tides do not have a similar effect of the width of the discharge or recharge zone.

### 3.8 Effect of PEMs on Darcy fluxes and hydraulic gradients across beach face

The effects of the PEMs on the water fluxes and hydraulic gradients across the beach face have been investigated for the homogeneous case. This means that it is mainly when the tide gets above 0.3 m (near high tide) that inflow will take place (excluding that due to density effects).
Feflow can compute the absolute value of the water (Darcy) flux \( q \) in any line segment given by the user, e.g., along the beach face. Since it is the absolute value, it tells nothing about direction. The gradient can be computed from Darcys law;

\[
q = K \times i
\]

where \( K \) is the hydraulic conductivity (here 25 m/d) and \( i \) is the gradient in the direction of flow. Thus in the present case;

\[
i = \frac{q}{K} = \frac{q}{25}
\]

With salt transport the hydraulic gradient \( i \) represents that of forced and free convection. Forced convection is caused by hydraulic gradients (any direction), free convection is caused by density differences (vertical).

Figure 27 shows the simulated salt distribution and flow field without PEMs at high tide. This figure corresponds to Figure 6.

![Simulated salt distribution and flow field](image)

Notice that there are two areas with inflow and an area in between with outflow. The areas with inflow have Darcy fluxes around 0.3 m/d (see velocity scale in legend), thus the inflow gradient is around 0.012. Figure 28 shows the situation at mean tide, when the tide is going from high to low tide (draining). Discharge is out of the beach right at MSL. The maximum outflow Darcy flux is around 1 m/d, thus the maximum outflow gradient is around 0.04. Figure 29 shows the situation at low tide. Flow is outwards mainly at the LWM. The maximum outflow Darcy flux and gradient is slightly smaller than at mean tide.
Figure 28 Simulated salt distribution and flow field for the homogeneous case without PEMs (mean tide) and a fixed head of 0.3 m at the right boundary.

Figure 29 Simulated salt distribution and flow field for the homogeneous case without PEMs (low tide) and a fixed head of 0.3 m at the right boundary.
Similar figures for the cases with PEMs are not shown simply because the velocities in the PEMs are so much higher than outside in the porous medium; instead refer to Figures 9 and 10. Recall that at high tide flow was outward across the beach face right at the three PEMs in the tidal zone due to the observed circulation around a PEM.

Figure 30 Absolute values of hydraulic gradients across beach face at low, mean and high tide. The gradient says nothing about direction.

Figure 30 shows the absolute values of the hydraulic gradients across the beach face at low, mean, and high tide. The gradients are calculated based on the simulated absolute values of the water fluxes at the beach face. Two sets of simulations are shown; solid and dashed lines are without and with PEMs, respectively. Recall that direction is not indicated, thus this figure can only be understood by also referring to Figures 27-29. In the cases of low and mean tide flow is always out, however at high tide flow is in and out. Referring to the high tide case with no PEMs, the two peaks at 8 m (LWM) and 33 m (HWM) correspond to the inflow shown in the figure above, and the peak in-between at around 20 m (MSL) is outflow. The gradients are highest in the cases of low and mean tide, around 0.35-0.04. The area of outflow tracks the receding water table very closely. However, notice that in the cases with PEMs (dashed lines) the two peaks are off-set by about 1-2 m. This is because the PEMs are located at 9 and 19 m, 1 m off the point where the low and mean water table cuts the beach. The PEMs therefore mainly redirects the point of maximum outflow during a receding tide. It is also seen that the PEM near the low tide line actually generates a higher gradient (and outflow) than in the case without a PEM. Otherwise, the simulated results are very alike. In the high tide case the two simulations are almost identical except that the three main active PEMs in the tidal zone (9, 19, and 29 m) generate outflow and not inflow due to the observed circulation. Thus, the not only are the gradients different, also direction.
3.9 Simulations with higher and lower inflow

Two other sets of simulations were carried out; (a) with a higher freshwater inflow using a fixed head of 1.5 m at the upstream boundary and (b) with a lower freshwater inflow using a fixed head of 0 m (=MSL). The two cases are compared with the base case (fixed head of 0.3 m) in Figure 31 for the situation of a homogeneous beach.

Freshwater inflow oscillates between 6.8-8 m$^2$/day in the higher inflow case and oscillates between -2.4 to +1.2 m$^2$/day in the lower inflow case. The high negative outflow in the last situation occurs because at high tide there is a significant head gradient inland.

Figure 31 demonstrates that the freshwater inflow has a significant impact on the saltwater distribution, here shown at high tide. With higher inflow the saltwater wedge is pushed back, while the lower inflow case results in significant buoyancy effects because a denser fluid overlies a lighter fluid that is almost stagnant.

Table 4 and 5 summarize the results. The most interesting cases are the high inflow case with a clay layer and all low inflow cases. When a clay layer is present in a high inflow case there is a significant relative extra inflow when the PEMs are present, see also Figure 25. Outflow is also higher when including PEMs. In absolute fluxes the outflow is much greater, 0.79 m$^2$/day, in comparison to only 0.22 m$^2$/day for inflow. Overall the PEMs therefore have a positive effect, on the order of 10%.

<table>
<thead>
<tr>
<th></th>
<th>Inflow (m$^2$/day) – difference in %</th>
<th>Outflow (m$^2$/day) – difference in %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No PEM</td>
<td>With PEM</td>
</tr>
<tr>
<td>Homogeneous</td>
<td>1.09</td>
<td>1.17</td>
</tr>
<tr>
<td>Gravel Layer</td>
<td>1.17</td>
<td>1.20</td>
</tr>
<tr>
<td>Clay Layer</td>
<td>0.72</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 4 Summary of simulation results with a higher freshwater inflow using a fixed head of 1.5 m.

In the low inflow case the presence of PEMs have a positive effect ranging from 9-29%. Again, when a clay layer is present there is a notable effect increasing drainage by 29%, while changes in inflow are small. This is interesting as the base case, Table 2, showed that the clay layer scenario only had a minor impact on the drainage effect of the PEMs. However, outflow is now very small, about half of the inflow.
Figure 31 Simulated salt distribution at high tides for three cases of freshwater inflow.
<table>
<thead>
<tr>
<th></th>
<th>Inflow (m$^2$/day) – difference in %</th>
<th>Outflow (m$^2$/day) – difference in %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No PEM</td>
<td>With PEM</td>
</tr>
<tr>
<td>Homogeneous</td>
<td>2.12</td>
<td>2.09</td>
</tr>
<tr>
<td>Gravel Layer</td>
<td>2.84</td>
<td>2.86</td>
</tr>
<tr>
<td>Clay Layer</td>
<td>1.71</td>
<td>1.78</td>
</tr>
</tbody>
</table>

Table 5 Summary of simulation results with a lower freshwater inflow using a fixed head of 0.0 m.

The effect of the PEMs on the hydraulic gradients across the beach face were investigated for the low inflow case (h=0.0 m at right boundary). When the tide rises from mean to high tide, inflow will be generated due to a gradient.

Figure 32 Simulated salt distribution and flow field for the homogeneous case without PEMs (high tide) and a fixed head of 0.0 m at the right boundary.
Figure 33 Simulated salt distribution and flow field for the homogeneous case without PEMs (mean tide) and a fixed head of 0.0 m at the right boundary.

Figure 32 shows the simulated salt distribution and flow velocities. Inflow takes place near the HWM. The maximum Darcy fluxes are around 1.5 m/d or $i=0.06$. Figure 33 shows the situation at mean tide. Density effects are greatest at this point. The flow field is rather complicated, where (dense) water either flows from the zone between MSL and HWM towards the right boundary or towards the MSL. Maximum Darcy flux is outward at MSL, but only half of that during high tide.

Figure 34 Simulated salt distribution and flow field for the homogeneous case without PEMs (high tide) and a fixed head of 0.0 m at the right boundary.
Figure 34 shows the situation at low tide. The salt water distribution affects the flow pattern below the tidal zone, but, generally flow is outward near the LWM, with a smaller gradient than at mean tide.

Figure 35 shows the calculated hydraulic gradient across the beach face. The (inflow) gradient at high tide is now higher than in the case with a high freshwater inflow, almost 0.06. The effects of an outward gradient near the three active PEMs are still seen, although the effect of the PEM at x=29 m is less due to the relative stronger inflow. The (outflow) gradients in the low and mean tide cases are a little lower than in the case with higher freshwater inflow. The same phenomena with PEMs generating outflow in the PEMs away from the water table line is still seen.

![Graph showing hydraulic gradients across beach face at low, mean and high tide. The gradient says nothing about direction.](image)

**3.10 Effect of depth of PEMs**

The effects of placing the PEMs 1 m deeper in the coastal aquifer was investigated for the homogeneous case.

Figure 36 shows the simulated hydraulic gradients, which again is compared to the situation without PEMs. The main difference is now at high tide. At low and mean tides the results are almost similar.
Figure 36 Absolute values of hydraulic gradients across beach face at low, mean and high tide for the case with PEMs located 1 m deeper. The gradient says nothing about direction.

The differences at high tide are caused by the circulation now taking place deeper in the coastal aquifer when the PEMs are located 1 m deeper. A new set of simulations were conducted with a refined mesh around the PEMs. Figure 37 and 38 show the results for the two situations near the PEM at the HWM; Figure 37 where the PEMs are located from 0.5-1.5 m below the beach face, and Figure 38, where they are located 1 m deeper.

The circulation pattern is clear in both cases, but in Figure 37 it results in flow outwards at the beach face. When the PEMs are located one meter deeper the flow is inward at the PEM. Thus, the observed differences in Figure 36 are primarily due to a change in flow direction.

If the two most inland PEMs were excluded then the two sets of simulations in Figure 36 were almost identical.
4. Conclusions

The effects of tides on groundwater flow and salt transport have been simulated in an idealized cross-section using the numerical model Feflow. The conceptual model was created in order to mimic the conditions at Holmsland on the West coast of Denmark (Engesgaard, 2006). Thus, the numerical model is two-dimensional. The model simulates variably-saturated flow with density-dependent flow and salt transport. Furthermore, it allows for a seepage condition at the tidal zone.

Several scenarios were simulated. The base case assumes an open system with a freshwater inflow corresponding to what might be realistic at the field site. Several other freshwater inflows were simulated with higher inflow and lower inflow. The numerical study compares the results without
and with PEMs in order to investigate the effects of including PEMs. The PEMs were simulated as pipes with a much higher hydraulic conductivity and were placed in the beach approximately according to how real PEMs are installed. It is only the slotted screen of a PEM that is included. The effect of having the PEM connect to a gravel layer or penetrate a lower-permeable layer was investigated. Finally, the effects of having a gravel layer connect to the sea or not were evaluated.

The numerical model still only resembles field conditions in an approximate way;

- It is a 2D model, which means that 3D flow phenomena around the PEMs (pipes) are not included. More importantly the pipes will over-represent the effects in the 2D model. The width of the model is implicitly assumed to be 1 m, while in reality the diameter of the pipes in the model is only 0.08 m.
- The discretization in the vertical direction (approximately 0.5 m) controls the thickness of the gravel/silt/clay layers. Whether these layers are thinner or thicker is not known.
- The hydraulic conductivity of the gravel layer in the base case is rather high. One may speculate whether this is a reasonable assumption given that small sand grains might fill up the pore space between the gravel yielding a hydraulic conductivity that is more comparable to that of sand. The hydraulic conductivity of the silt/clay layers are realistic.
- The PEMs all connect with the layers, which of course is not necessarily the case in a real situation.

The following observations are made;

- There is a complicated flow field in the tidal zone, where the water table detaches from the beach and drops to below the PEMs. In almost all cases this results in a tidal response that is not symmetric, i.e., the observed fluxes are not the same at the two low/high tides during one tidal cycle. This could be the result of this non-linear behaviour of the water table dropping and rising along the length of the PEMs. The PEMs are active one by one as the tide rolls past a PEM. It was observed that the PEMs always carried water upward due to a circulation pattern where water enters the PEMs near the bottom and exits at the top. This seems intuitively correct at low tide, however it also occurs at high tide, where flow is also downward. This downward flow seeps into the PEMs and flows vertically upward.
- The PEMs have an effect on the salt distribution near the PEMs in the tidal zone. The PEMs transport a lot of water, for example approximately 50% of that discharging to the sea at low tide in the base case. However, this is not the same as saying that they increase discharge with this amount, because in reality the just move water to another area with sand or gravel. Flow inside the PEMs are of the order 0.2-0.3 cm/s.
- The PEMs allow water and salt to flow slightly more rapidly into and out of the beach. In most cases they have an effect both ways sometimes resulting in a negative effect of the PEMs, sometimes in a positive effect. Despite this, the effects (positive or negative) are generally small when compared with the integrated outward or inward flux during a tidal cycle. In the base case (homogeneous beach) the extra outflow caused by the PEMs is on the order of 5%. Only in the case with a very low freshwater inflow are the changes in in- and outflow higher than this.
- There seems to be a positive effect of the PEMs in the case where a clay layer is present (and connected to the sea and thus acting as a low-permeable barrier to flow). This effect is most pronounced in the high and low freshwater inflow case, while in the base case it is less apparent.
• The PEMs do not appear to increase the width of the discharge zone significantly. Only in the case of steady flow is the width increased from 1-2 m to 2-3 m.
• The effects of changing the hydrogeological conditions (gravel/silt/clay layer, freshwater inflow) have a larger impact on inflow and outflows across the beach than having PEMs or not.
• The depth of the PEMs has an effect on the fluxes/gradients at the beach face. If they are close to the beach face the circulation pattern means that flow is generally outward at the beach face. If they are located 1 m deeper flow the circulation pattern does not influence the gradients at the beach face and flow is inward.

5. References